Question 1

a) Convert 15° to radians in terms of π.
b) Find a primitive of 0.
c) Solve $3x^2 - 11x - 4 = 0$
d) Simplify $6 - \frac{x-2}{4}$.
e) Rationalise the denominator $\frac{2}{\sqrt{3}+2}$.
f) Graph the solution of $|x+4| < 2$ on a number line.
g) If $x = 9$ and $y = \frac{1}{16}$, evaluate $x^{\frac{1}{2}} y^{\frac{-1}{2}}$.

Question 2 (Start a new page)

a) Differentiate:
   i) $\tan x$
   ii) $(x^2 + 1)^3$
   iii) $\cos(2x + 1)$
b) Evaluate:
   i) $\int_0^{\frac{1}{3}} \frac{1}{3x+1} dx$
   ii) $\int_0^2 \frac{e^x - 1}{e^x} dx$
c) If $y = e^{3x}$ show that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

Question 3 (Start a new page)

A parallelogram has two of its sides along the lines $y = x$ and $y = 2x$. The vertex which does not lie on either of these two lines has coordinates (6,8).
i) Draw a diagram illustrating the above information.
ii) Show that one of the other sides of the parallelogram is the line whose equation is $y = x + 2$.
iii) Find the equation of the line that is the fourth side of the parallelogram.
iv) Find the other two vertices of the parallelogram.
v) Find the perpendicular distance from (6,8) to $y = x$.
vi) Find the area of the parallelogram.

Question 4 (Start a new page)

a) Given α and β are the roots of the equation $x^2 - 8x + 5 = 0$ find the value of:

i) $\alpha + \beta$.

ii) $\alpha\beta$.

iii) $\alpha^2 + \beta^2$.

b) The line $x + 2y - 3 = 0$ forms chord AB on the parabola $y = x^2 - 2$. Find the x coordinate of the midpoint of AB.

c) Solve the inequality $x > 6 - x^2$.

d) Find the least value of $x^2 + x - 6$.

Question 5 (Start a new page)

i) Find the two stationary points of the function $y = x(x-4)^2$ and determine their nature.

ii) Sketch the curve.

iii) Find the equation of the tangent at $x = 2$ on the curve.

iv) Show that the tangent meets the curve again at (4,0).

Question 6 (Start a new page)

a) i) Draw the graph of $y = \sin 2x$ for $0 \le x \le \pi$.

ii) Find the number of distinct roots of $\sin 2x = \frac{1}{4}$ for $0 \le x \le \pi$. Justify your answer.

b)

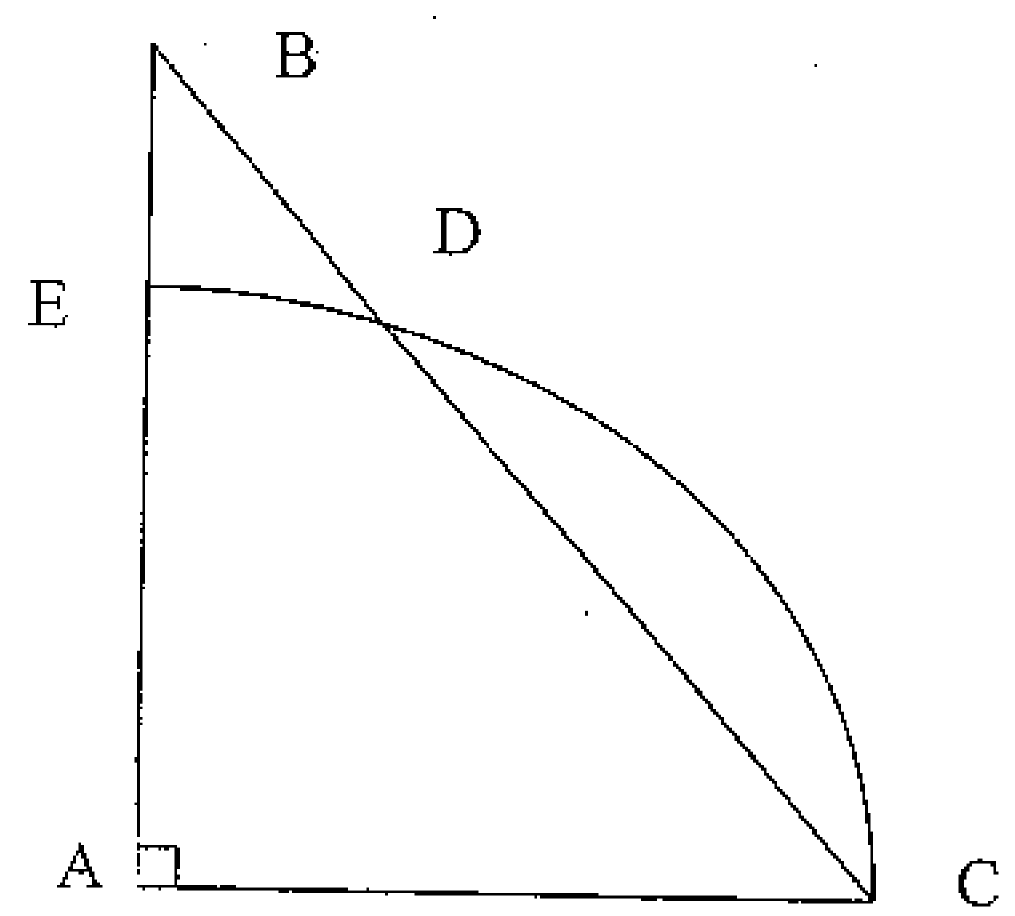


DIAGRAM NOT TO SCALE

A triangle ABC has a right angle at A. The angle at C is 60°. The side AC is 2cm long. A circular arc, centre A, radius 2cm cuts the side BC at D and the side AB at E.

i) Explain why AD = 2cm.

ii) Prove ΔACD is equilateral.

iii) Prove ΔABD is isosceles.

iv) Find the area of the portion BED.

Question 7 (Start a new page)

a) In one bag there are 2 red and 4 yellow balls, while in a second bag there are 3 red balls and 6 yellow balls. One of the bags is selected at random and two balls withdrawn, with no replacement of the first ball before the second is withdrawn. What is the probability that the balls are:

i) Both red?

ii) Both the same colour?

b) A man borrows \$30000 and agrees to repay it over 10 years. Interest is calculated at 0.6% per month and is charged monthly.

i) Show that the amount owing after n months is $A_n = 30000(1.006)^n - M(1 + 1.006 + 1.006^2 \ldots\ldots\ldots 1.006^{n-1})$ where M = monthly repayments.

ii) Find the monthly repayment to the nearest dollar.

c) Consider $0.3\dot{9}\dot{5}$

i) Express it as an infinite series.

ii) Hence express it as a fraction in simplest form.

Question 8 (Start a new page)

a) Evaluate:

i) $\int 2\sec^2 3x\,dx$

ii) $\int_1^2 \frac{x^2 - 2x + 1}{x}\,dx$

b) Using Simpson's rule with 3 function values, find an approximation of $\int_1^5 \ln x.dx$.

c) i) Find the derivative of $x\ln x - x$ with respect to x.

ii) Using i) find the exact value of $\int_1^5 \ln x.dx$.

Question 9 (Start a new page)

a) A particle moves so that at time t seconds its velocity v m/s is given by $v = 2\pi + 2\pi \sin \pi t$, ($t \geq 0$). When $t = 0$, $x = -2$:

i) Find an expression for the acceleration at time t.

ii) Find the time when the particle is first stationary, and its displacement at that time.

b) An electric current flowing in a circuit, left to itself, falls at a rate proportional to it's value i.e. $\frac{dI}{dt} = -kI$, where I = current.

i) If $I = I_0 e^{-kt}$ (I_0 and k are constants, t is time in seconds) show that I satisfies the differential equation $\frac{dI}{dt} = -kI$.

ii) If the current drops to half its original value in $\frac{1}{4}$ sec find:

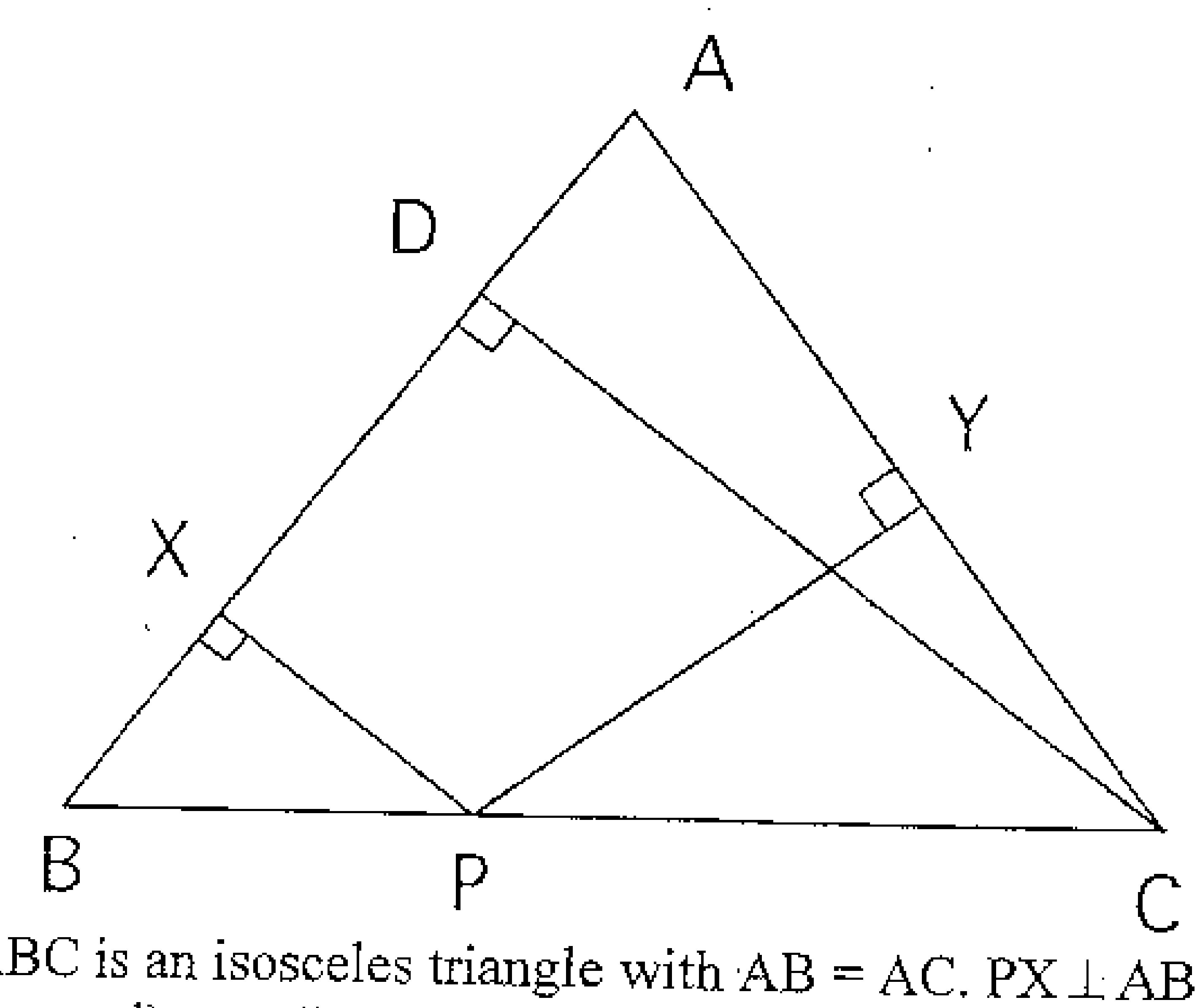
I) The value of k to four decimal places.

II) How long an instrument will continue to give a reading if the instrument can just detect a current one millionth part of the original. Answer to one decimal place.

Question 10 (Start a new page)

a) Find all real solutions of x which satisfy the equation $x^4 = 7x^2 - 12$.

b)



ABC is an isosceles triangle with AB = AC. PX $\perp$ AB, CD $\perp$ AB and PY $\perp$ AC.

i) Reproduce the diagram onto your exam paper.

ii) Prove ΔPXB $|||$ ΔCDB.

iii) Prove ΔCDB $|||$ ΔPYC.

iv) Hence show that PX + PY = CD.

END OF PAPER

Sol^{ns} 2U JRAHS TRIAL 2000.

Question 1.

(a) $\frac{15\pi}{180}$ 1

(b) c (constant) 1

(c) $3x^2 - 11x - 4 = 0$

$(3x+1)(x-4) = 0$ 2

$x = -\frac{1}{3}, \ 4$

(d) $6 - \frac{x-2}{4}$

$= \frac{24 - (x-2)}{4}$ 2

$= \frac{26-x}{4}$

e) $\frac{2}{\sqrt{3}+2}$

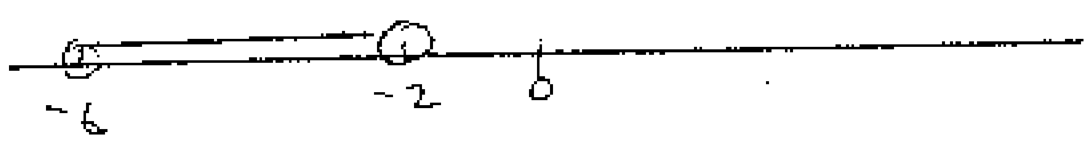
$= \frac{2}{\sqrt{3}+2} \cdot \frac{\sqrt{3}-2}{\sqrt{3}-2}$ 2

$= -(2\sqrt{3}-4)$

f) $|x+4| < 2$

$x+4 < 2 \ \cap \ x+4 > -2$

$x < -2 \ \cap \ x > -6.$ 2



g) $9^{\frac{1}{2}} \cdot 16^{-\frac{1}{2}}$

$= 3 \cdot 4$ 2.

$= 12.$

Question 2

(a)(i) $\frac{d}{dx}\tan x = \sec^2 x$ 1

(ii) $\frac{d}{dx}(x^2+1)^3 = 3(x^2+1)^2 \cdot 2x$ 1

$= 6x(x^2+1)^2$

(iii) $\frac{d}{dx}\cos(2x+1) = -2\sin(2x+1)$ 1

(b)(i) $\int_0^{\frac{1}{3}} \frac{1}{3x+1}\,dx = \frac{1}{3}\Big[\ln(3x+1)\Big]_0^{\frac{1}{3}}$ 3

$= \frac{1}{3}\ln 2$

(ii) $\int_0^2 \frac{e^x - 1}{e^x}\,dx = \int_0^2 (1 - e^{-x})\,dx$

$= \Big[x + e^{-x}\Big]_0^2$ 3

$= 1 + e^{-2}$

(c) $y = e^{3x}$

$y' = 3e^{3x}$ 3

$y'' = 9e^{3x}$

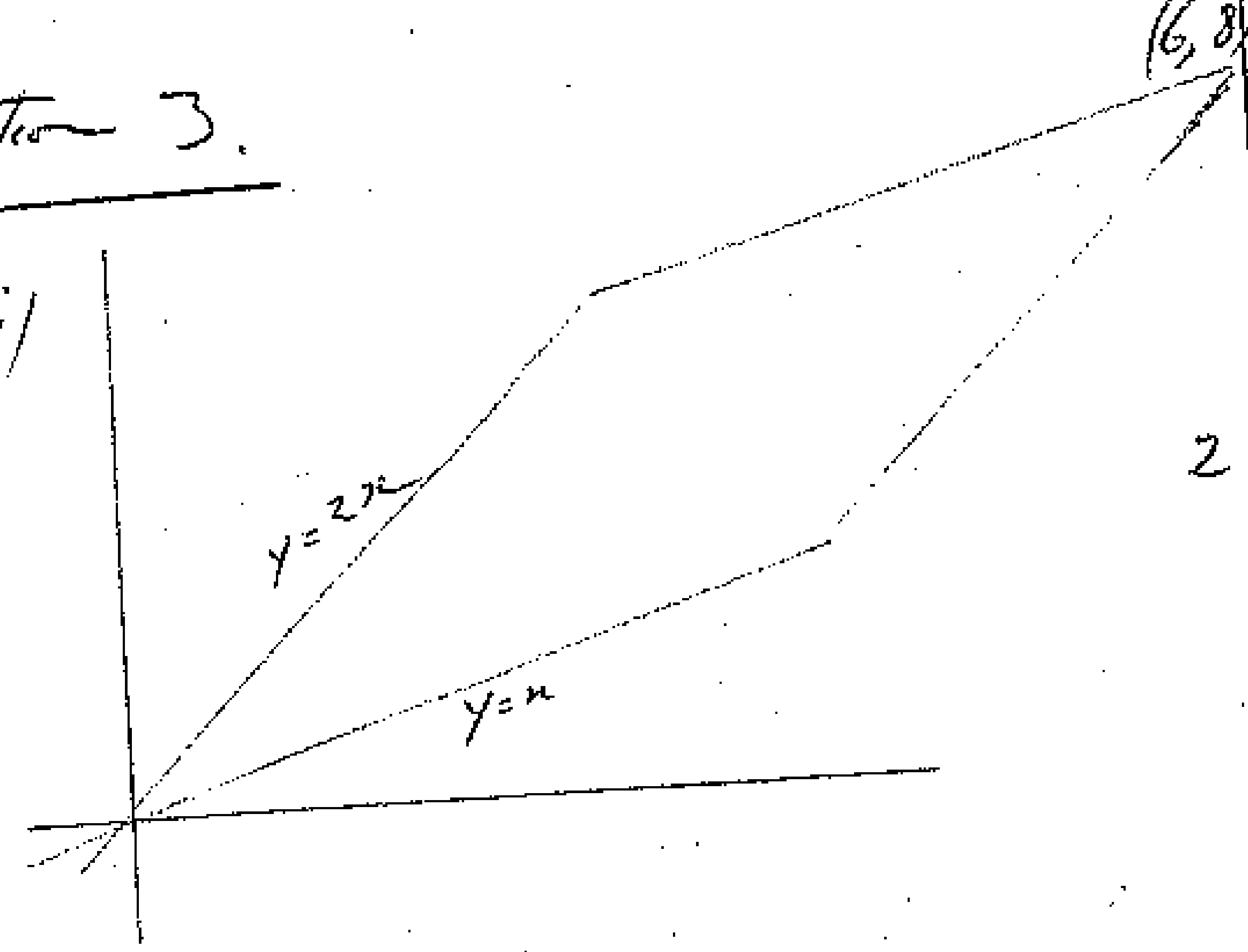
$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 9e^{3x} - 15e^{3x} + 6e^{3x}$

$= 0$

1st 2U TRIALS 2000 (Cont)

Question 3.

(i)



2

(ii) $(y-8) = 1(x-6)$ 2

$y = x+2$

(iii) $y-8 = 2(x-6)$ 2

$y = 2x - 12 + 8$

$y = 2x - 4$

(iv) $\left.\begin{array}{l} y = 2x \\ y = x+2 \end{array}\right\}$ $\quad$ $y = x$, $y = 2x-4$ 2.

$x+2 = 2x$ $\quad$ $2x - 4 = x$

$x = 2$ $\quad$ $x = 4$

$\therefore y = 4$ $\quad$ $y = 4$

$\therefore$ Vertices $(0,0), (2,4), (6,8), (4,4)$

(v) Perp dist $= \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$ $\quad$ $x_1 = 6$, $y_1 = 8$, $a = 1$, $b = -1$

$= \left| \dfrac{2-4}{\sqrt{2}} \right|$ 2

$= \sqrt{2}$ u.

(vi) Area of PARM $= \sqrt{32} \cdot \sqrt{2}$ 2

$= 8$ u^2

Question 4.

(a)(i) $\alpha + \beta = 8$ 1

(ii) $\alpha\beta = 5$ 1

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 64 - 10$ 2

$= 54$

(b) $y = x^2 - 2$ $\quad$ $y = \dfrac{-x+3}{2}$

Solving $\quad -x + 3 = 2x^2 - 4$ 3

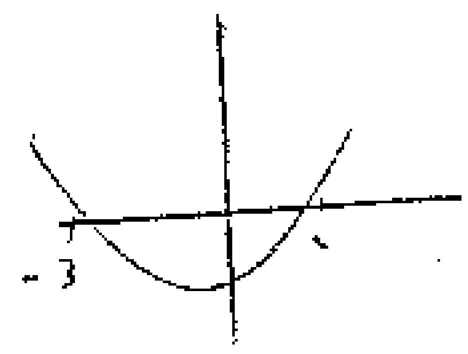
$2x^2 + x - 7 = 0$

x Coord of M point $= \dfrac{\alpha+\beta}{2} = -\dfrac{1}{4}$

(c) $x > 6 - x^2$ 3

$x^2 + x - 6 > 0$

$x > 2, \ x < -3$



(d) From diagram 2

$\left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 6$

$= \frac{1}{4} - \frac{1}{2} - 6$

$= -6\frac{1}{4}$

uestion 5

$y = x(x-4)^2$

$y' = (x-4)^2 + x.2(x-4)$

$= (x-4)(3x-4)$ 2

$y'' = (x-4).3 + (3x-4)$

$= 6x - 16$ 1

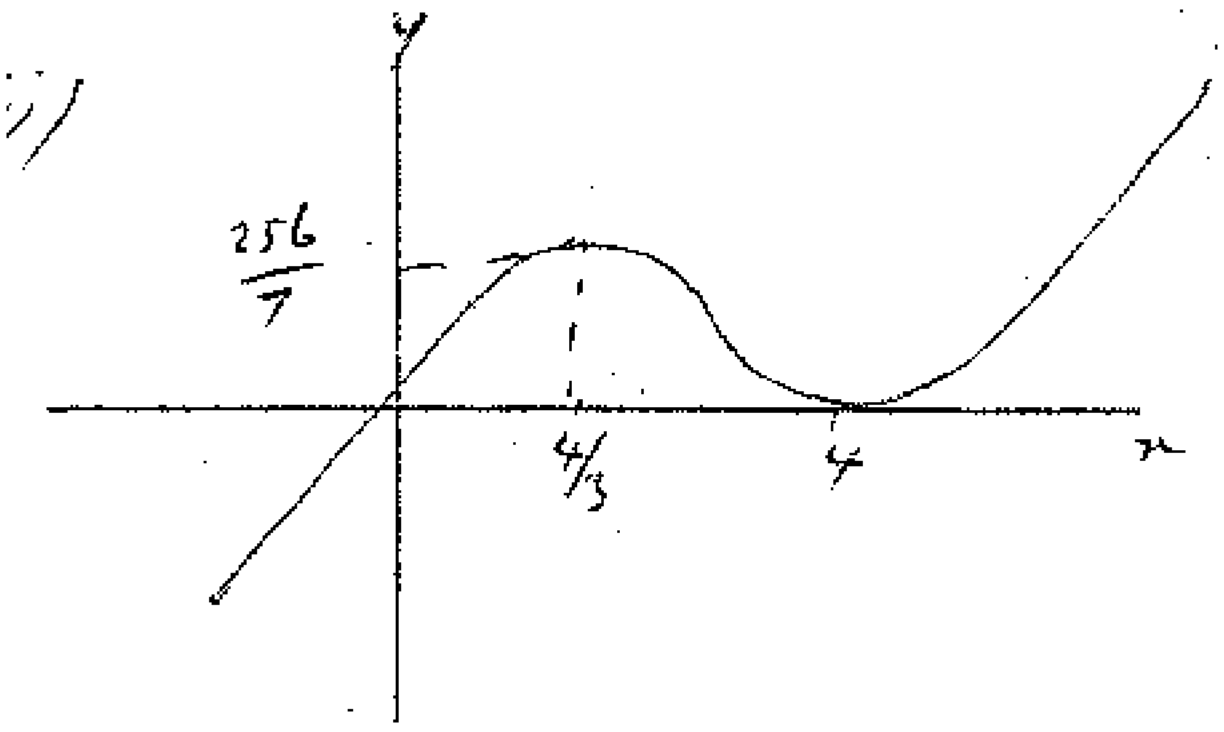
i) STAT pts $(4,0)$ + $\left(\frac{4}{3}, \frac{256}{27}\right)$ 2

ad int 2nd deriv.

$x = 4 \quad \frac{d^2y}{dx^2} > 0 \quad \therefore$ Min 1

$x = 4/3 \quad \frac{d^2y}{dx^2} < 0 \quad \therefore$ Max 1

ii)



1

i) when $x = 2 \quad y = 8$

Slope at $x = 2$ is -4 3.

$\therefore$ Tangent $y - 8 = -4(x-2)$

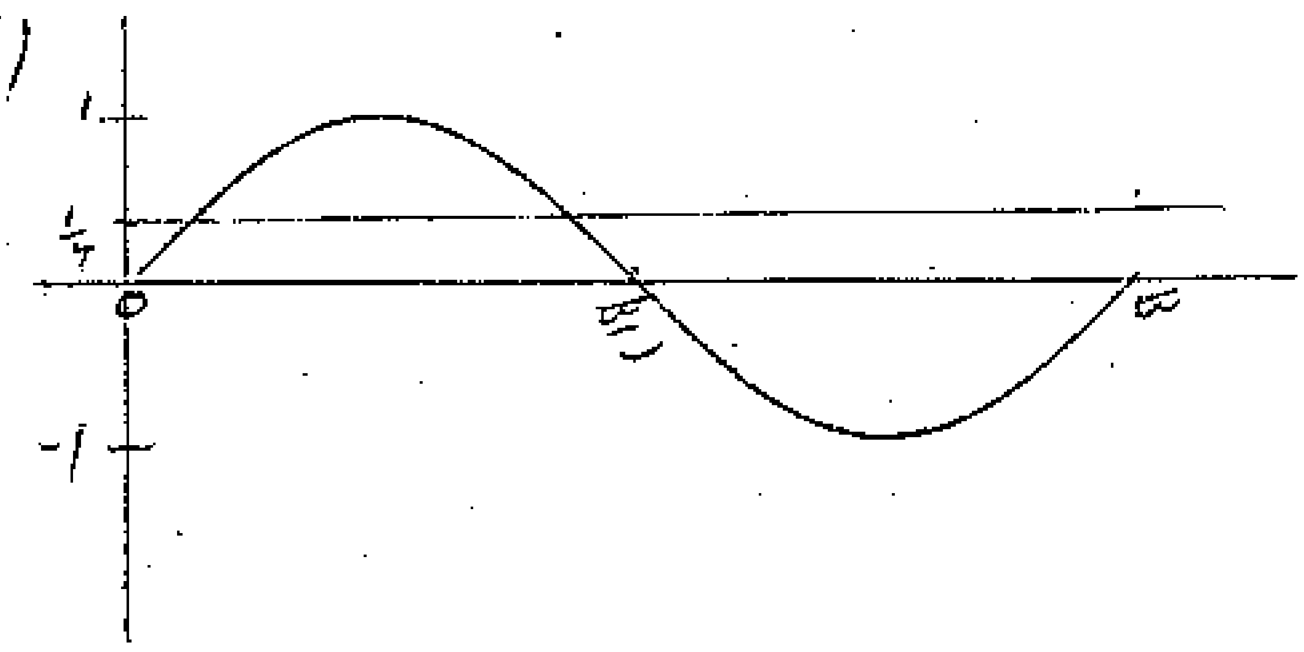
$y + 4x - 16 = 0$

) $(4,0)$ satisfies both 1

$y + 4x - 16 = 0$ and $y = x(x-4)^2$.

Question 6.

(a) (i) 1



2

(ii) line + curve intersect twice
$\therefore$ 2 roots.

(b)(i) radius of circular arc 1

(ii) $\triangle ACD$ has equal sides $AD = AC$

$\therefore \hat{ADC} = \hat{ACD} = 60°$ .. equal angles opposite equal sides. 2

$\therefore \triangle ADC$ is equilateral $\left(\text{two angles} = 60°\right)$

(iii)

In $\triangle DAB$, $D\hat{A}B = 30°$ - - - $90° - 60°$

$\hat{ABC} = 30°$ angle sum of $\triangle ABC$.

$\therefore \triangle ABD$ is isosceles $\left(\text{base angles} = 30°\right)$

(iv) Area of Sector $ADE = \frac{1}{2}r^2\theta$

$= \frac{1}{2} . 4 . \frac{\pi}{6}$ 2

$= \pi/3$

Area of portion $BED = \sqrt{3} - \pi/3$ 1

(Area of $\triangle ABD = \frac{1}{2} . 2 . 2 \sin 120°$)

1

Question 7

(i) $\frac{1}{2}\left(\frac{2}{6}\cdot\frac{1}{5} + \frac{3}{9}\cdot\frac{2}{8}\right)$ 2

$\frac{1}{30} + \frac{1}{24}$

$= \frac{3}{40}$

(ii) $\frac{3}{40} + \frac{1}{2}\left(\frac{4}{6}\cdot\frac{3}{5} + \frac{6}{9}\cdot\frac{5}{8}\right)$ 2

$= \frac{3}{40} + \left(\frac{1}{5} + \frac{5}{24}\right)$

$= \frac{29}{60}$

(b) (i) $A_1 = 30000 \times 1.006 - M$

$A_2 = (30000 \times 1.006 - M)1.006 - M$

$= 30000 \times 1.006^2 - M(1 + 1.006)$

$A_n = 30000 \times 1.006^n - M(1 + 1.006 + \dots + 1.006^{n-1})$

(ii) Let $A_n = 0$.

$30000 \times 1.006^n = M(1 + 1.006 + \dots 1.006^{n-1})$ 4.

$= M\left(\frac{1(1.006^{120} - 1)}{0.006}\right)$

$M = \$351$ (nearest dollar)

(c) $0.39595 \dots$ 1

i) $0.3 + .095 + .00095 + \dots$

ii) $\frac{3}{10} + \frac{0.095}{1 - .01} = \frac{3}{10} + \frac{95}{990}$ 2

$= \frac{297 + 95}{990}$

$= \frac{392}{990}$

Question 8.

(a) i) $\int 2\sec^2 3x \, dx$ 1

$= \frac{2\tan 3x}{3} + C$

ii $\int_1^2 \frac{x^2 - 2x + 1}{x} \, dx$ 3

$= \int_1^2 \left(x - 2 + \frac{1}{x}\right) dx$

$= \left[\frac{x^2}{2} - 2x + \ln x\right]_1^2$

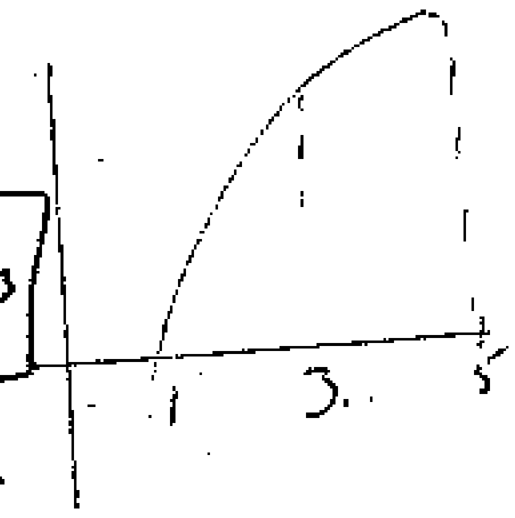
$= (2 - 4 + \ln 2) - \left(\frac{1}{2} - 2\right)$

$= -2 + \ln 2 + \frac{3}{2}$

$= -\frac{1}{2} + \ln 2.$

(b)

$\int_1^5 \ln x \, dx \doteq \frac{2}{3}\left[(\ln 5 + 0) + 4\ln 3\right]$



$\doteq \frac{2}{3} \times 6.0038$ 3.

$\doteq 4.002$

c (i) $\frac{d}{dx}(x\ln x - x) = 1 + \ln x - 1$

$= \ln x.$

(ii) $\therefore \int_1^5 \ln x = \left[x\ln x - x\right]_1^5$

$= (5\ln 5 - 5) - (-1)$

$= 5\ln 5 - 4$

$= 4.047$

Question 9.

(a) i) $a = 2\pi^2 \cos \pi t$ 1

ii) $x = 2\pi t - 2\cos \pi t + c$ 1

$t=0 \quad x=-2 \quad \therefore c=0$

$x = 2\pi t - 2\cos \pi t$ 1

1^{st} Stationary when $-2\pi = 2\pi \sin \pi t$ 1

$\sin \pi t = -1$

$t = 3/2$ 1

Displacement when $t = 3/2 = 3\pi$. 1.

(b)(i) $I = I_0 e^{-kt}$ 1

$\frac{dI}{dt} = -kI_0 e^{-kt}$

$= -kI$ since $I = I_0 e^{-kt}$

(ii)(I) $I = I_0 e^{-kt}$ 1

$\frac{1}{2} = e^{-0.25k}$ 1

$k = 2.7726$ 1

II $\frac{1}{1000000} = e^{-2.7726 t}$ 1

$t = 5.0$ min (1 Dec place) 1

Question 10

(a) $x^4 - 7x^2 + 12 = 0$

$(x^2-4)(x^2-3) = 0$ 2

$x = \pm 2, \pm\sqrt{3}$

b. (ii) $B\hat{X}P = B\hat{D}C$ ... given 2

$\hat{B}$ is common

$\therefore \triangle PXB \ ||| \ \triangle CDB$ ... (AA)

(iii) $A\hat{B}C = A\hat{C}P$ ... equal angles opposite equal sides (AB = AC)

$C\hat{D}B = P\hat{Y}C$ ... Both 90° given

$\therefore \triangle CDB \ ||| \ \triangle PYC$ ... (AA).

(iv) $\frac{PX}{CD} = \frac{BP}{BC}$ ... Corresponding sides $\triangle PXB \ ||| \ \triangle CDB$ 2

$\frac{PY}{CD} = \frac{CP}{BC}$ ... Corresponding sides $\triangle PYC \ ||| \ \triangle CDB$ 2

$PX + PY = \frac{BP}{BC} \cdot CD + \frac{CP}{BC} \cdot CD$

$= CD\left(\frac{BP+CP}{BC}\right)$ 2

$= CD.$